

COSMIC STRINGS ARE CURRENT-CARRYING.

Anne-Christine DAVIS^{1,2} and Patrick PETER^{2,3}

¹*CERN - Theory division, CH-1211 Geneva 23, Switzerland,*

²*D.A.M.T.P., Silver Street, Cambridge CB3 9EW, England,*

³*D.A.R.C. (UPR176, CNRS), Observatoire de Paris, 92195 Meudon, France.*

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A synthesis of previous work done on the microscopic structure of cosmic strings in realistic models is made and reveals that strings are expected to be not only superconducting in the sense of Witten, but also generically current-carrying, either at the GUT scale or at the electroweak scale. This applies to any GUT string forming model leading to the standard electroweak theory as a low energy limit. The current consists of charged vector bosons. Cosmological consequences are briefly discussed.

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I. INTRODUCTION

Most cosmological applications of cosmic strings [1,2] have been essentially based on noncurrent-carrying vortices, namely those having no internal structure and hence describable by means of the simple Goto-Nambu action. They would have appeared as topological defects during a very early phase transition such as predicted by many Grand Unified Theories (GUT). They have been shown to form a network that could explain the large scale structure of the universe [2] and the temperature fluctuations in the cosmic microwave background radiation [2]. Because of their origin in particle physics models and their high predictive power (only one free parameter, namely the energy per unit length U), they represent the major opponent to the almost standard inflationary scenario [3].

Cosmic strings of the current-carrying type, such as proposed by Witten [4], have recently received a revival of interest in particular thanks to their potential usefulness in producing the observed baryon number asymmetry [5]. Following this original attempt [4] to show that currents could appear along string's worldsheets, a number of particle physics models were shown (actually designed for that purpose) to similarly exhibit strings endowed with superconducting properties. Hence the question of how generic this feature may be. The purpose of this letter is to address this question. We show that within the framework of the standard GUT paradigm with the electroweak model as a low energy limit, it is inconceivable that a phase transition in the early universe produced strings that would still be structureless, as the ones mostly used in any numerical simulation.

Currents in strings change our understanding of the string cosmology in particular because the scaling property of the network might well be absent, or modified, in such models. Thus, it sets various constraints on the underlying particle physics models on scales otherwise inaccessible. In fact, as we shall see later, because of the so-called vorton problem [6], there are already cases

which are ruled out even though only very little is currently known about current-carrying cosmic string cosmology. Such a strong consequence makes it clear that this new branch of cosmology has been largely underestimated until now and deserves much more attention. Among many other possibilities, let us just mention for instance here the obvious one that primordial magnetic fields might be generated. This could fairly well lead to a complete revision of our views concerning large scale structure formation.

The reason why currents must be expected to be present in cosmic strings is twofold. First, the existence of a symmetry restoration region [7] and then that of spontaneous current generation [8]; both are generic phenomena in particle physics models predicting cosmic strings. As we shall see, these mechanisms are responsible for the appearance of a current first, and then for its stability. This yields three distinct possibilities for the energy scales involved, namely, using the notations of Ref. [9], the mass scale m at which the string forms (i.e., with $U \sim m^2$) and m_* being characteristic of the current intensity. The careful analysis that follows reveals that only a few numerical values for these string's parameters are actually realizable in Nature, namely $m \sim m_* \sim \eta_{\text{GUT}} \sim 10^{15}$ GeV the scale of Grand Unification, $m \sim \eta_{\text{GUT}}$ and $m_* \sim \eta_{\text{EW}} \sim 100$ GeV the scale of electroweak symmetry breaking, or an intermediate scale for m provided it exceeds $\eta_{\text{int}} \sim 10^{10}$ GeV, where the bound is set up by vorton formation, still with $m_* \sim \eta_{\text{EW}}$, or finally similar scales $m \sim m_*$, given then [6] by $\eta_v \sim 10$ TeV.

This work is organized as follows: in section 2, we set the problem and the notation; in section 3, we summarize the basics of symmetry restoration around a cosmic string, then section 4 is devoted to the spontaneous current generation mechanism while section 5 uses both effects to yield the conclusion that cosmic strings are expected to be current-carrying independently of the underlying particle physics model.

Finally, we briefly discuss cosmological consequences

including the corresponding constraints due for instance to the vorton problem.

II. THE PROBLEM

Let us first fix the notation that is used throughout. For the sake of generality, we shall consider a theory, effective or actual, with the following scheme of symmetry breaking:

$$G \xrightarrow{\Phi} H \xrightarrow{h} \dots \rightarrow SU(3) \times SU(2) \times U(1) \xrightarrow{H} SU(3) \times U(1), \quad (1)$$

where G might already be the result of previous symmetry breaking(s) (hence it is not necessarily simple), and Φ , h and H stand for the various Higgs fields responsible for the phase transitions (arrows) over which they are symbolized on Eq. (1), meaning for instance that H is the ordinary $SU(2)$ doublet of the standard electroweak model. Note also that nothing prevents h to be identical to either H or Φ and what we clearly request in fact is solely that there are at least two symmetry breakings, one of them being the electroweak one. Moreover, we shall assume $\pi_1[G/SU(3) \times U(1)] \not\sim \{0\}$ so that strings are formed at some stage.

Let us now turn more specifically to the string forming model. The Lagrangian density we are interested in is the GUT one without fermions, namely

$$\mathcal{L} = \frac{1}{2}(D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2}(D_\mu h)^\dagger (D^\mu h) - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - V(\Phi, h), \quad (2)$$

where

$$D_\mu \Phi \equiv (\partial_\mu + igT_a A_\mu^a)\Phi, \quad D_\mu h \equiv (\partial_\mu + ig\tau_a A_\mu^a)h, \quad (3)$$

are the covariant derivatives of the Higgs fields Φ and h , the former transforming under G according to the representation given by the T_a 's, basis for the Lie algebra $\mathcal{L}(G)$, and the latter according to the representation given by the τ_a 's [note in particular that h transforms trivially under most of the generators of G except for those that span the subalgebra $\mathcal{L}(H)$]. The kinetic term for the gauge vectors A_μ^a follows from the definition

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c, \quad (4)$$

where g is the (potentially running) coupling strength and the f^{abc} 's are the structure constants of the gauge group G with

$$[T_a, T_b] = if^{abc}T_c. \quad (5)$$

Note also that we are using a metric with signature $+2$ when writing the Lagrangian (2).

A vortex solution to the Euler-Lagrange equations derived from Eq. (2) is given classically by [10]

$$\Phi = f(r) \exp(-in\theta T_s) \Phi_0, \quad (6)$$

$$T_a A_\mu^a = -\frac{n}{gr} v(r) \delta_{\mu\theta} T_s, \quad (7)$$

for a string aligned along the z -axis in rectangular polar coordinates. T_s is the string's generator and the boundary conditions are

$$f(0) = v(0) = 0, \quad \lim_{r \rightarrow \infty} f(r) = \lim_{r \rightarrow \infty} v(r) = 1,$$

Φ_0 being the actual Higgs field's constant configuration that minimises the potential at infinity. The scale is set by its vacuum expectation value (VEV) $|\Phi_0|^2 = m^2$ following the notation of the introduction.

To end this section, and before turning our attention to the actual symmetry restoration mechanism, let us consider the various terms that generically will be present in the Lagrangian (2). In particular, there should be direct coupling terms of the form $|\Phi|^2 |h|^2$, given that they are renormalisable. Indeed, such a term must be present, would it be only at the one-loop level, because the number of broken generators $\dim(G) - \dim(H)$ is, as can be checked by direct evaluation of all possible cases of Lie algebras and their maximal subalgebras [11], far greater than the rank of G . Thus, the gauge bosons coupled to Φ , in any gauge, call them $A_\mu^{(\phi)}$, must be coupled to some gauge bosons coupled to h , $A_\mu^{(h)}$ say. Therefore, $\exists T^{(\phi)} \in \mathcal{L}(G)$ and $\exists T^{(h)} \in \mathcal{L}(H)$; $[T^{(\phi)}, T^{(h)}] \neq 0$. So the corresponding structure constants $f^{(\phi)(h)(h)}$ and $f^{(h)(\phi)(\phi)}$ do not all vanish and the coupling exists. Similarly, it can be shown that the other interesting term coupling h and the string's generator cannot be made to vanish; thus, h carries T_s hypercharge. We shall call X_μ the gauge vector boson associated with T_s .

Both the two terms discussed in the previous paragraph are required for a charged-coupled current to exist in the string's core as we shall now see. Since we have just shown that they are generically present, this achieves our proof of the current-carrying abilities of cosmic-strings.

III. SYMMETRY RESTORATION

A generic feature of cosmic strings formed at a scale m is that they lead to restoration of lower energy symmetries. For example, a GUT scale string restores the electroweak symmetry around it in a region proportional to the electroweak scale [7]. Obviously in this region the electroweak particles are massless. Similarly, if there are a series of phase transitions, the GUT string will restore these out to a region depending on the scale of the phase transition. To be concrete, let us consider a string with gauge field X_μ . The string's field couples to the Higgs

field h of the less symmetric sector as we previously discussed,

$$D_\mu h = (\partial_\mu - \frac{i}{2}\tilde{g}\vec{\tau} \cdot \vec{W}_\mu - \frac{i}{2}\alpha X_\mu)h, \quad (8)$$

where the coupling strength α can either be a direct coupling – as expected in GUTs – or induced by loop corrections and we have specifically extracted out the part depending on the string’s generator X_μ from the other ones which we called \vec{W}_μ . (Recall that α and \tilde{g} have in general no reason to be identical to g unless the A_μ^a ’s representation choice is set up for this to occur.) In either case, the term (8) is, again, generic. Putting in the Nielsen-Olesen form for X_μ [12] for a unit winding number string as given in Eq. (7), one solves the equations of motion for the electroweak theory using the ansatz [7] $h = e^{i\ell\theta}\eta(r)$. In particular, the presence of the extra term in Eq. (8) changes the h and part of the \vec{W} equations of motion – in the background (7) it is energetically favourable that there is a part of \vec{W} that vanishes.

Calling Z (in analogy with the electroweak model [7]) that part of \vec{W} which does not vanish, one finds that in order to ensure finite energy at large distances, h and Z must have profiles similar to that of the Nielsen-Olesen [12] profile. Thus, using a trial solution [7]

$$h = \frac{m_\star}{2} \begin{cases} (r/r_1)^{a/2} & r < r_1 \\ 1 & r > r_1 \end{cases}, \quad (9)$$

$$GZ = GZ_\theta = -a \begin{cases} r/r_1^2 & r < r_1 \\ 1/r & r > r_1 \end{cases}, \quad (10)$$

where G is an effective coupling constant that can be calculated out of the choice of τ_a ’s (for instance in the case of h being identical to H , the Higgs doublet of the electroweak model, $G^2 = g^2 + g'^2$, with g and g' the gauge couplings of $SU(2)_L$ and $U(1)_Y$), m_\star is the VEV of the Higgs field h , and

$$a = 2\ell + \frac{\alpha}{g}\Theta(r - m^{-1})$$

with Θ the Heaviside step function.

Minimizing the energy integral, one finds that $m_\star r_1 \sim 1$ and the energy of the trial solution is am_\star^2 . Thus, there is a region of H symmetry restoration around the G scale string of order the H scale. We also note that if α is large, then it is energetically favorable for the Higgs field h to wind around the string, i.e., for $\ell \neq 0$. The previous discussion indeed applies to any intermediate scale that would be restored in an analogous fashion because nothing of what has just been stated is explicitly dependent on the symmetry breaking scheme and the Higgs structure. In particular, it can be applied (and it is in fact in this context that this mechanism was originally derived [7]) to the case of a GUT string coupled to the electroweak sector, i.e., with h identified with the doublet H and $m_\star \sim M_W \sim 100$ GeV.

This point being settled, let us now turn to the second necessary ingredient.

IV. SPONTANEOUS CURRENT GENERATION

The reason why cosmic strings are generically superconducting and in fact even current-carrying is because of the spontaneous current generation mechanism, which proceeds as follows. Whenever the mass of a charged (or hypercharged, provided the corresponding generator remains massless) vector boson gets lowered in some finite size region surrounding a string’s core (e.g., the region of symmetry restoration), then it becomes energetically favourable for this charged boson to acquire a nonvanishing VEV in the string’s core, thereby effectively breaking spontaneously the Lorentz symmetry along the string’s worldsheet. Thus, one just has to prove that in general such a charged boson exists to prove generic superconductivity of cosmic strings. The three distinct regimes mentioned earlier are then made apparent once a close examination of the various cases is made. Let us first discuss in more detail the current generation mechanism itself.

Suppose the Higgs field h to be coupled to a massless hypercharge (like the actual electric charge) carrying vector boson C_μ in the sense that at least part (but possibly all) of its mass originates from h ’s VEV. Then close to the string’s core, C_μ is less massive than far from it because of the symmetry restoration. Now the field equations for C_μ near the vortex, where $\Phi = h = 0$, read as a part of the general A_μ^a field equations

$$\nabla^\mu F_{\mu\nu}^a - g f^{abc} A^{b\mu} F_{\mu\nu}^c = 0, \quad (11)$$

which is usually solved in vacuum by the trivial solution $A_\mu^a = 0$, and so $C_\mu = 0$. However, this solution is nothing but a gauge choice and it is possible to choose anything else without, at least in vacuum, changing the total energy of the configuration. But we are near a string where gradients of scalar fields are important since $\Phi = 0$ at $r = 0$ is topologically required while bounding the overall energy dynamically requires $|\Phi|^2 = m^2$ at $r \rightarrow \infty$. As a result, changing the solution of Eq. (11) is not just a gauge choice and the configuration having the minimum energy is one having nonzero C_μ ’s VEV [13].

The conclusion here is that any hypercharge carrying field next to the string may fluctuate and thus lead to a source term in Eq. (11) which in turn is used in the Higgs field equations

$$D_\mu D^\mu \Phi = -2 \frac{\delta V}{\delta |\Phi|^2} \Phi, \quad D_\mu D^\mu h = -2 \frac{\delta V}{\delta |h|^2} h, \quad (12)$$

through the covariant derivatives. It turns out [13] that the only necessary condition for this charging-up to spontaneously occur is just that the corresponding hypercharge be massless. This is where the various energy scales are involved since many cases are possible: the first is that the charged vector bosons mixing the quarks and the leptons at the GUT scale acquire a mass at or right after the string-forming symmetry breaking. In

this case, the massless hypercharge is simply the electric charge and the current amplitude is of the order $m_* \sim m \sim \eta_{GUT}$. The same applies also for the W^\pm bosons of the electroweak sector (in this case h is identified with H). These bosons are known to exist, at an energy scale $m_* \sim \eta_{EW}$ that must be less than m , since we have not observed any string-forming phase transitions yet. Thus, cosmic strings, if they exist, must at least carry an electroweak current. Finally, there is the possibility that another phase transition occurs at an intermediate energy scale, in which case the corresponding currents might fairly well be charged coupled or neutral because it is sufficient to build up the current that the hypercharge be massless only over a finite region of space, but not necessarily over all of it. Thus, seen from the outside, such a current-carrying string would be of the neutral kind [14].

V. CONCLUSIONS

By putting together previous results on the internal structure of cosmic strings, we have shown that they become current-carrying, at least at the electroweak scale. This is because the electroweak symmetry is restored around the string [7], giving rise to spontaneous current generation due to charged vector bosons condensing in the string [8,13]. If there are appropriate intermediate scales in the theory, then analogous results hold. However, this is not the only way a string might acquire spontaneous gauge vector current. Charged bosons coupling quarks and leptons are also perfectly reasonable candidates for that purpose. In fact, it entirely depends on the actual scale at which the string-forming symmetry breaking occurs.

One cosmological consequence needs to be emphasized at this point: in the case where $m \sim m_* \sim \eta_{GUT}$, one ends up with current-carrying cosmic strings whose internal structure, i.e. the equation of state relating the energy per unit length and the tension, looks much like [8] that derivable for the charged coupled Witten model [4,15]. But using Carter's formalism [16] to evaluate the stability [17] of vorton remnants, it was found [18] that such an equation of state always leads to some stable states. Since the current-carrying state represents a minimum of the total energy, they are quantum mechanically stable as well. Thus such strings are ruled out and necessarily constrain the corresponding particle physics models.

It had previously been shown [6,19] that the existence of vortons as stable remnants of superconducting strings severely constrains the underlying particle theory. In Ref. [19], it was also shown that for the universe to be radiation dominated at nucleosynthesis, GUT-scale strings must not produce vortons at temperatures above $\sim 10^6$ GeV. Similarly for intermediate-scale strings becoming superconducting at the same phase transition (i.e., with $m \sim m_*$) then that scale m must be less

than 10^{10} GeV [19] whilst a less conservative estimate gives this to be an upper bound of $m \lesssim 10^4$ GeV only (the articles by B. Carter in Refs. [6]). However, our generic results have shown that cosmic strings become superconducting at the electroweak scale. In this case, the constraints are rather different and two regimes need to be considered. If strings are in the scaling regime by the time of the electroweak phase transition, then they are sufficiently dilute (~ 1 per horizon volume) that vorton remnants are harmless. If however, they are in the friction dominated regime, and hence copious, then the above constraints apply. Strings produced at scales $m \gtrsim 10^{10}$ GeV are in the scaling regime by the electroweak phase transition. Cosmological consequences of GUT strings becoming superconducting at the electroweak phase transition are currently under investigation and will be discussed in a forthcoming publication.

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